Mark Scheme (Results)

January 2017

Pearson Edexcel International A Level in Mechanics 3 (WME03/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
1		
	Vol $=(\pi)\int y^2\mathrm{d}x=(\pi)\int_0^4 9(4-x)\,\mathrm{d}x=9(\pi)\left[4x-\frac{1}{2}x^2\right]_0^4$	M1
	$=9(\pi)(16-8)=(72(\pi))$	A1
	$\pi\int_0^4 9x(4-x)\,\mathrm{d}x=9\pi\left[2x^2-\frac{1}{3}x^3\right]_0^4$	M1A1
	$=9\pi\left(32-\frac{64}{3}\right)\ (=96\pi)$	dM1
	$\bar{x}=\frac{96\pi}{72\pi}=\frac{4}{3}$ (Accept 1.3 or better)	dM1A1cao
		[7]

M1 Attempting the required vol integral, with or without π Power to increase by 1 in at least one term and neither to decrease. Limits not needed.

A1 Correct result after substitution, with or without π provided same as previous line; need not be simplified.

M1 Attempting $(\pi)\int y^2 x\,\mathrm{d}x$; with or without π, limits not needed

A1 Correct integration and correct limits seen; with or without π provided same as previous line.

dM1 Substitute their limits, with or without π provided same as previous line; need not be simplified. This may be implied by subsequent working. Depends on second M mark

dM1 Dividing the results of their integration. π to be in both numerator and denominator or in neither. First 2 M marks needed.

A1cao Correct answer, exact or decimal

NB Inclusion of ρ should be treated in the same way as π ie ignore for the separate integrals but must be in both integral or neither for the 4th M mark.

Question Number	Scheme	Marks
2		
(a)	$0.6a = \left(3t + \frac{1}{2}\right)$	
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{5}{3}\left(3t + \frac{1}{2}\right)$	M1
	$v = \frac{5}{2}t^2 + \frac{5}{6}t \quad (+c)$	
	$(t = 0, v = 0 \;\Rightarrow c = 0) \qquad \therefore v = \frac{5}{2}t^2 + \frac{5}{6}t$	A1 (2)
(b)	$\frac{10}{3} = \frac{5}{2}t^2 + \frac{5}{6}t$	M1
	$3t^2 + t - 4 = 0$	
	$(3t+4)(t-1) = 0$	
	$t = 1$ at A	dM1A1
	$s = \left[\frac{5t^3}{6} + \frac{5t^2}{12}\right]_0^1, \quad = \frac{5}{4}\text{m} \quad$ oe	M1, A1 (5)
		[7]

(a)

M1 Form an equation of motion with the acceleration in form dv/dt. Must include the mass.

A1 Integrate and complete. Award for a correct expression even if no reference has been made to the constant of integration.

(b)

M1 Set their expression equal to 10/3

dM1 Factorise (or use formula or completing the square) to reach t = ... (May include a second value.)

A1 Correct value for t. (Ignore extra answer provided limits on following integral are correct.)

M1 Integrate the expression for v, limits needed, lower limit must be 0 (seen or implied), their t for upper limit.

A1 Substitute correct limits to obtain 5/4 or 1.25 m

Indefinite integration in (b) - last 2 marks:
M1 for the integration with or without the constant, A1 for the correct answer.

Question Number	Scheme	Marks
3	ratio of masses $\quad 96\pi a^3\rho \quad 18\pi a^3\rho \quad 78\pi a^3\rho$	M1A1
	$\quad\quad\quad\quad\quad 16 \quad\quad 3 \quad\quad 13$ oe	
	Distance from O: $\quad 3a \quad \frac{9a}{8} \quad \bar{x}$	B1
	$16\times 3a - 3\times\frac{9a}{8} = 13\bar{x}$	M1A1ft
	$\bar{x} = \frac{357}{104}a = 3\frac{45}{104}a$ or $3.432a$....(accept $3.4a$ or better)	A1 [6]

M1 Attempt the ratio of masses. Terms to be consistent in use of π, a^3, ρ and formulae must be correct.
A1 Correct ratio, any equivalent form. Simplification not required.
B1 Correct distances from *O* or any other point
M1 Form a moments equation using their mass ratio and distances. Must be dimensionally correct and Hemisphere must be subtracted on LHS.
A1ft Correct equation ft their mass ratio and distances.
A1 Correct answer, exact or decimal. Must be from *O* and must be positive.

Question Number	Scheme	Marks
4 (a)	$T_a \sin\theta = mg + T_b \sin\phi$	M1
	$\frac{3}{5}T_a = T_b \sin\phi + mg$	A1A1
	$T_a \cos\theta + T_b \cos\phi = m \times 4a\omega^2$	M1
	$\frac{4}{5}T_a + T_b \cos\phi = 4ma\omega^2$	A1A1
	$\frac{3}{5}T_a + \frac{4}{5}T_a = mg + 4ma\omega^2$ (inc use of $\cos\phi = \sin\phi$)	dM1
	$T_a = \frac{5}{7}m\left(4a\omega^2 + g\right)$ *	A1cso (8)
(b)	$\frac{1}{\sqrt{2}}T_b = 4ma\omega^2 - \frac{4}{7}m\left(4a\omega^2 + g\right)$	M1
	$T_b = \sqrt{2}\left(\frac{12}{7}ma\omega^2 - \frac{4}{7}mg\right)$ oe $\left(\text{eg } \frac{4\sqrt{2}}{7}m\left(3a\omega^2 - g\right)\right)$	A1 (2)
(c)	$T_b \geqslant 0 \Rightarrow 3a\omega^2 \geqslant g$	M1
	$\omega \geqslant \sqrt{\frac{g}{3a}}$ $k = 3$	A1 (2)
		[12]

(a)M1 Resolve vertically. Both tensions to be resolved.

A1 Both tension terms correct. θ, ϕ allowed

A1 Completely correct equation. $\sin\theta$ replaced with 3/5 now or later, $\sin\phi, \sin 45°$ or $1/\sqrt{2}$ accepted

M1 NL2 along radius. Both tensions to be resolved. Accel in either form.

A1 Both tension terms correct, θ, ϕ allowed.

A1 Completely correct equation. $\cos\theta$ replaced with 4/5 now or later, $\cos\phi, \cos 45°$ or $1/\sqrt{2}$ accepted. Acceleration to be $4a\omega^2$ now.

dM1 Eliminate T_b between the 2 equations. Dep on both previous M marks and use of $\cos\phi = \sin\phi$ or numerical values for all trig functions. There must be evidence of the solution of the pair of equations.

A1cso Given result obtained from correct working.

(b)

M1 Obtain T_b by any valid means. If sub T_a in one equation the given expression (or equivalent) must be used.

A1cao Correct expression for T_b in any equivalent form.

NB One or both marks for (b) can be gained for using the given expression for T_a in an equation obtained in (a), but watch for "circular" arguments which obtain both tensions this way. Marks for the equation used can only be awarded for work shown in (a)

(c)

M1 Setting their $T_b \geqslant 0$ and deducing an inequality connecting ω^2 (or ω) and g

A1 Completing to the required form with a correct value for k (need not be shown explicitly)

Question Number	Scheme	Marks
5(a)	$F \leqslant \mu 4mg$	B1
	$F = T = \frac{3mg}{l} \times \frac{1}{3}, \leqslant 4\mu mg$	M1,A1ft
	$\mu \geqslant \frac{1}{4}$ $\ast$	A1cso (4)
(b)	(Assume string returns to its natural length)	
	$\text{EPE} = \frac{3mgl^2}{2l}$	B1
	Work done against friction $= \frac{2}{5} \times 4mg \times l$	B1
	$\frac{1}{2} \times 4mv^2 = \frac{3mgl^2}{2l} - \frac{2}{5} \times 4mg \times l$	M1A1ft
	$v^2 \leqslant 0 \quad \Rightarrow$ string does not become slack.	A1 cso (5) [9]
	(Alternative: Assume extension is x when particle comes to rest)	
	$\text{EPE lost} = \frac{3mgl^2}{2l} - \frac{3mgx^2}{2l}$	B1(either)
	Work done against friction $= \frac{2}{5} \times 4mg \times (l - x)$	B1
	$\frac{3mgl^2}{2l} - \frac{3mgx^2}{2l} = \frac{8}{5} mg(l - x)$	M1A1ft
	$x = \frac{1}{15} l, \ (x = l)$ pos ext at $x = \frac{1}{15} l$ $\therefore$ at rest before string becomes slack	A1cso

(a)B1 Correct inequality or equation provided clear this is maximum friction.
M1 Resolve horizontally including use of HL (correct formula) and an attempt at the extension (ie not l) Mass can be m or $4m$
A1ft Correct **inequality** follow through their max friction
A1 Obtain GIVEN inequality with no errors seen.
If = used throughout and inequality only at final stage, B1M1 only unless they convince you.
NB If it is not clear that max friction is being used, only M mark is available.
(b)B1 Correct EPE when extension is l
B1 Correct WD against friction
M1 Attempt energy equation with KE, EPE and WD terms. EPE term to be of the form $k \frac{\lambda x^2}{l}$, mass m or $4m$.
A1ft Correct equation, follow through their EPE and WD terms. All signs to be correct.
A1cso Solving for v^2 or stating $v^2 \leqslant 0$ (from a fully correct equation) and giving the conclusion. No errors anywhere in the working.
ALT For last 3 marks: Show WD to nat length > initial EPE (M1A1) Correct conclusion (A1)
ALT B1 Either of the 2 required EPE terms correct
B1 Correct WD against friction
M1 Attempt energy equation with a difference of EPE terms and a WD term. EPE terms to be of the form $k \frac{\lambda x^2}{l}$
A1ft Correct equation, follow through their EPE and WD terms. All signs to be correct.
A1cso Solve for x and state conclusion. No errors seen

Question Number	Scheme	Marks
6 (a)	$2mg = \frac{20mge}{5l}$	M1
	$e = \frac{l}{2}$	A1
	$AB = 5.5l$ oe	A1ft (3)
(b)	$2mg - T = 2m\ddot{x}$	M1
	$2mg - \frac{20mg\left(0.5l + x\right)}{5l} = 2m\ddot{x}$	dM1A1A1
	$\ddot{x} = -\frac{2g}{l}x \quad \therefore$ SHM	A1 cso (5)
(c)	$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{2g}} = \pi\sqrt{\frac{2l}{g}}$ oe	M1A1ft (2)
(d)	$a\omega = \frac{1}{5}\sqrt{gl} = a\sqrt{\frac{2g}{l}}$	M1A1ft
	$a = \frac{l}{5\sqrt{2}}$ oe	A1 (3)
(e)	$BD = \frac{1}{2}a$	
	$-\frac{1}{2}a = a\cos\omega t$	M1
	$\cos t\sqrt{\frac{2g}{l}} = -\frac{1}{2}$	A1
	$t = \sqrt{\frac{l}{2g}}\cos^{-1}\left(-\frac{1}{2}\right)$	dM1
	$t = \sqrt{\frac{l}{2g}} \times \frac{2\pi}{3} = \frac{2\pi}{3}\sqrt{\frac{l}{2g}}$	A1cso (4)
		[17]

1

Question Number	Scheme	Marks

(a)	
M1	Resolve vertically including using Hooke' law to find the tension. Formula for HL to be correct.
A1	Solve their equation to find the equilibrium extension
A1ft	Add $5l$ to their equilibrium extension to obtain AB
ALT	Use of extension $\left(AB-l\right)$
M1	Resolve vertically inc HL with extension $\left(AB-l\right)$ Formula for HL to be correct.
A1	Fully correct equation
A1	$AB=5.5l$
(b)	
M1	Resolve vertically (at a general point). HL not needed for this mark; acceleration can be $\ddot{x}$ or a. Mass $2m$ or m
dM1	Use HL with extension $\left(0.5l+x\right)$ or $(e+x)$ (provided e is clearly defined to be the equilibrium extension by their work in (a)). Dependent on the previous M mark.
A1	Correct difference of forces either way round; acceleration can be $\ddot{x}$ or a.
A1	Fully correct equation with acceleration $\ddot{x}$.
A1cso	Correct equation starting $\ddot{x}=\ldots$ and conclusion.
(c)	
M1	Use Period $=\dfrac{2\pi}{\omega}$ with their ω from an "SHM" equation (ie $\ddot{x}=\pm\omega^2 x$ or $a=\pm\omega^2 x$)
A1ft	Correct period follow through their ω
(d)	
M1	Use $v=a\omega$ or $v^2=\omega^2\left(a^2-x^2\right)$ with $x=0$ later, their ω
A1ft	Correct equation, follow through their ω
A1cao	Correct amplitude
(e)	
M1	Use $x=a\cos\omega t$ or $x=a\sin\omega t$ with their ω from an "SHM" equation (ie $\ddot{x}=\pm\omega^2 x$ or $a=\pm\omega^2 x$) and $x=\pm\dfrac{1}{2}a$ Equivalents in terms of l acceptable.
A1	Correct equation (limited ft) Allow their amplitude in terms of l (if used) but ω to be correct.
dM1	Solve to $t=\ldots$, where t is the required time. Must be in radians. This must be a complete (correct) method, so if eg $x=a\sin\omega t$ and/or $x=\dfrac{1}{2}a$ used there must be work included to obtain the required answer from their equation.
A1cso	Correct time, as shown or equivalent. (cso applies to part (e) work only.)

1

Question Number	Scheme	Marks
7 (a)	$\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = mgr(1+\cos 60)$	M1A1A1
	$R - mg = m\frac{v^2}{r}$	M1A1A1
	$R - mg = 2mg \times \frac{3}{2} + m\frac{U^2}{r}$	dM1
	$R = 4mg + m\frac{U^2}{r}$	A1 (8)
(b)	P leaves the bowl with speed U at 60° to the horizontal	
	Vert speed $= U\sin 60 \quad \left(= \frac{U\sqrt{3}}{2}\right)$	B1
	$0 = (U\sin 60)^2 - 2gs$	M1
	$s = \frac{3U^2}{8g}$	A1
	Greatest height $= \frac{3}{2}r + \frac{3U^2}{8g}$	A1ft (4)
(c)	$0 = U\sin 60t - \frac{1}{2}gt^2$	M1
	Time to level of rim $= \frac{2U\sin 60}{g} \quad \left(= \frac{U\sqrt{3}}{g}\right)$	A1
	Horiz speed $= U\cos 60 \quad = \left(\frac{1}{2}U\right)$	
	Horiz dist $= \frac{2U^2\sin 60}{2g}$	B1
	$U^2 > 2gr \quad \therefore$ horiz dist $> \frac{4rg\sin 60}{2g} = 2r\sin 60$	M1
	$\therefore$ P does not fall back into the bowl	A1cso (5) [17]
ALT 1	**Alternatives for (c)** **Work to the highest point:**	
	$0 = U\sin 60 - gt, \quad t = \frac{U\sqrt{3}}{2g}$	M1,A1
	Horiz dist to highest point $= \frac{U^2\sqrt{3}}{4g}$	B1
	$U^2 > 2gr \quad \therefore$ horiz dist $> r\sin 60, \quad \therefore P$ does not fall back into the bowl	M1,A1cso

Question Number	Scheme	Marks
ALT 2	**Find the vertical height when "above" A**	
	time to $A = \dfrac{2r\sqrt{3}}{U}$	B1
	Vert dist $= U\dfrac{\sqrt{3}}{2}\times\dfrac{2r\sqrt{3}}{U}-\dfrac{1}{2}g\dfrac{4r^2\times 3}{U^2}\ \left(=3r-\dfrac{6gr^2}{U^2}\right)$	M1A1
	$U^2 > 2gr\ \ \therefore$ vert dist >0, $\quad\therefore P$ does not fall back into the bowl	M1,A1cso

(a)

M1 Attempt an energy equation from A to B with 2 KE terms and a PE term (which must include a trig function.

A1 Correct KE terms (must be a difference)

A1 Correct PE term and all signs correct

M1 Attempt NL2 along the radius at B, acceleration in either form. If the equation at a general point is given this mark is not earned until the equation at B is reached.

A1 Correct difference of forces

A1 Correct mass x accel term and completely correct equation

dM1 Eliminate v between the two equations. Dependent on both previous M marks

A1 Correct expression for R. Any equivalent form accepted but must be positive.

(b)

B1 Correct vertical speed on leaving the bowl

M1 Use $v^2 = u^2 + 2as$ with their initial vertical speed (or any other complete method) to find the greatest height of P above the rim of the bowl.

A1 Solve to obtain the correct greatest height above the rim

A1ft Add $\dfrac{3r}{2}r$ to their previous answer to obtain the greatest height above the floor.

Energy methods:

(i) Using vertical speed only: marks exactly as above.

(ii) If initial speed U is used, account must be taken of the horizontal energy at the top.
B1 for the correct horizontal speed (or KE) M1 for the equation with correct number of terms A1A1ft as above. Attempts which do not include the horizontal energy will score 0/4 (apart from exceptional cases where the horizontal or vertical speed has been found correctly and B1 is earned)

(c)

NB There is no mark in (c) for the vertical speed. The mark in (b) **cannot** be awarded in (c)

M1 Use $s = ut + \dfrac{1}{2}at^2$ with their initial vertical speed and $s = 0$ to find the time until P returns to the level of the rim.

A1 Correct time obtained

B1 Correct horizontal distance. Allow with one, two trig functions or none

dM1 Compare their horizontal distance with their diameter of the rim using the inequality. Depends on the previous M mark

A1cso Correct conclusion from correct work

Question Number	Scheme	Marks

	Alternatives for (c)
ALT 1	Find the horizontal distance to the highest point
M1	Use $v = u + at$ with their $v = 0$ to find the time until P reaches it's highest point OR use any other complete method
A1	Correct time found
B1	Correct horizontal distance. Allow with one, two trig functions or none
dM1	Compare their horizontal distance with the radius of the rim using the inequality
A1cso	Correct conclusion from correct work
ALT 2	Find the vertical height when above *A*:
B1	Use horizontal motion to find the time to *A*
M1	Find the vertical height "above" *A*
A1	Correct height "above" *A*
dM1	Use the inequality determine whether or not *P* is above the rim
A1cso	Correct conclusion from correct work
NB	If the limiting value of U is assumed $\left(\text{ie } U = \sqrt{2gr}\right)$ M1A1B1 can be awarded for correct work. The final 2 marks are awarded only if consideration of the projectile motion is used to explain why *P* falls outside. eg increasing the initial speed will increase the (horizontal) range (M1)and correct conclusion from correct work (A1)

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